A Community Based Mobility Model for Ad Hoc Networks Research

ABSTRACT

Validation of mobile ad hoc network protocols relies almost exclusively on simulation. The value of the validation is, therefore, highly dependent on how realistic the movement models used in the simulations are. Since there is a very limited number of available real traces in the public domain, synthetic models for movement pattern generation must be used. However, most widely used models are currently very simplistic, their focus being ease of implementation rather than soundness of foundation. As a consequence, simulation results of protocols are often based on randomly generated movement patterns and, therefore, they may differ considerably from those that can be obtained deploying the system in real scenarios. Movement is strongly affected by the needs of humans to socialise or cooperate, in one form or another. Fortunately, humans are known to associate in particular ways that can be mathematically modelled and that have been studied in social sciences for many years. It is in fact undeniable that social relationships definitely bias movement patterns.

In this paper we propose a new mobility model that is founded on social network theory. The model allows collections of hosts to be grouped together in a way that is based on social relationships among the individuals. This grouping is only then mapped to a topographical space, with movements influenced by the strength of social ties that may change in different periods of the day. We have validated our model with real traces by showing that the generated mobility traces can be considered a very good approximation of human movement patterns. We have also run simulations of AODV and DSR using this mobility model and show how the delivery ratio is affected by this type of mobility.

1. INTRODUCTION

The definition of realistic mobility models is one of the most critical and, at the same time, difficult aspects of the simulations of applications and systems designed for mobile environments. Currently, there are very few and very recent publicly available data banks capturing node movement in real large-scale mobile ad hoc environments. For example, researchers at Intel Research Laboratory in Cambridge and the University of Cambridge distributed Bluetooth purpose-made devices to a certain number of people in order to collect data about human movements to study the characteristics of the co-location patterns among people. These experiments were firstly conducted among students and researchers in Cambridge [6] and then among the participants of InfoCom 2005 [16]. Other similar projects are the Wireless Topology Discovery project [24] at the University of California at San Diego and the campus-wide WaveLan traffic measurement and analysis exercises that have been carried out at Dartmouth College [12]. At this institution, a project with the aim of creating a repository of publicly available traces for the mobile networking community has also been started [21].

Until now, real movement traces have been rarely used for the evaluation and testing of protocols and systems for wireless networks, with the only exception of [39] and [15], in which the authors used respectively the movement traces collected from a campus scenario and direct empirical observations of the movements of pedestrians in downtown Osaka as a basis of the design of their models.

In general, synthetic models have been largely preferred [5]. The reasons of this choice are many. First of all, the available data are few: the academic and industrial projects providing publicly available data have started very recently. Second, these traces are related to very specific scenarios and it is still quite difficult to generalize their validity. However, as we will discuss later in the paper, these data show surprising common statistical characteristics, such as the same distribution of the duration of the contacts and inter-contacts intervals. Third, the available traces do not allow for sensitivity analysis of the performance of the algorithm, since it is not possible to vary the values of the parameters that characterize the simulation scenarios, such as the distribution of the speed or the density of the hosts. Finally, in some cases, it may be important to have a mathematical model that underlines the movement of the hosts in simulations, in order to study its impact on the design of protocols and systems.

Many mobility models for the generation of synthetic traces have been presented (a survey can be found in [5]). The most widely used of such models are based on random individual movement; the simplest, the Random Walk Mobility Model (equivalent to Brownian motion), is used to represent pure random movements of the entities of a system [7]. A slight enhancement of this is the Random Way-Point Mobility Model [18], in which pauses are introduced between changes in direction or speed. More recently, a large number of more sophisticated mobility models for ad hoc network research have been presented like [4, 17, 22].

However, all synthetic movement models are suspect because it is quite difficult to assess to what extent they map reality. It is not difficult to see, even only with empirical observations, that the random mobility models generate behaviour that is most unhuman-like. This analysis is confirmed by the examination of the available real traces. As we will discuss later in this paper, mobility models based on random mechanisms generates traces that show properties (such as the duration of the contacts between the mobile nodes and the inter-contacts time) that are very distant from those extracted from real scenarios.

Our work is based on a simple observation. In mobile ad hoc networks, mobile devices are usually carried by humans, so the movement of such devices is necessarily based on human decisions and socialization behaviour. It is, for instance, important to model the behaviour of individuals moving in groups and between groups, as clustering is likely in the typical ad hoc networking deployment scenarios of disaster relief teams, platoons of soldiers, etc. In order to capture this type of behaviour,we define models for group mobility that are heavily dependent on the structure of the relationships among the people carrying the devices. Existing group mobility models fail to capture this social dimension [5].

Within the emerging field of sensor networks, mobile hosts are not necessarily carried directly by humans. However, sensor networks are usually embedded in artefacts (such as cars or planes or clothing) or are spread across a geographical area (such as environmental sensors). In the former case, the movements of the sensors embedded in a car or in aeroplane, for instance, are not random but are dependent on the movements of the carriers; in the latter, movement is not generally a major issue.

Taken together, for those systems in which mobility is important and for which a synthetic mobility model is an essential ingredient, it would appear to be important to consider the influence of the human-level social network as something that informs likely individual and group mobility patterns. Fortunately, in recent years, such networks have been investigated in considerable detail, both in sociology and in other areas, most notably mathematics and physics. Mathematical models of such networks have been empirically shown to be useful in describing many types of relationships, including real social relationships [31, 28].

In this paper, we propose a new mobility model that is founded on social network theory, because this has empirically been shown to be useful as a means of describing human relationships. In particular, one of the inputs of the mobility model is the social network that links the individuals carrying the mobile devices. The model allows collections of hosts to be grouped together in a way that is based on social relationships among the individuals. This grouping is only then mapped to a topographical space, with topography biased by the strength of social ties. We will also show that the movements of the hosts are also driven by the social relationships among them. The model also allows for the definition of different types of relationships during a certain period of time (i.e., a day or a week). For instance, it might be important do be able to describe that in the morning and in the afternoon of weekdays, relationships at the workplace are more important than friendships and family one, whereas the opposite is true during the evenings and weekends.

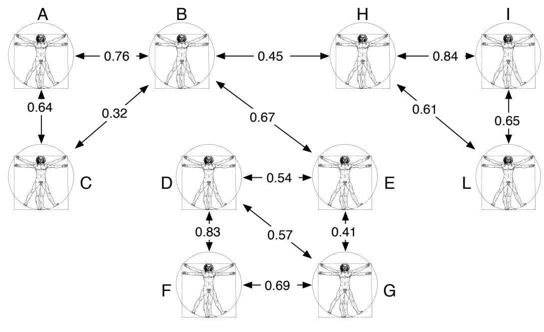

Figure 1: Example of social network.

We evaluate our model using real mobility traces provided by Intel Research Laboratory in Cambridge and we show that the model provides a good approximation of real movements in terms of some fundamental parameters, such as the distribution of the contacts duration and inter-contacts time. In particular, the data shows that an approximate power law holds over a large range of values for the inter-contacts time. Contacts duration distribution, instead, follows a power law for a more limited range. These characteristics of distribution are also very similar to those observed by the researchers at the University of California at San Diego and Dartmouth College.

The proposed model is partially based on the work presented by Musolesi et alii in [25]. With respect to that paper, many aspects of the model have been revised to try to map reality with more accuracy. More specifically, in this work the formation of the groups is based on an algorithm for the detection of communities in social networks [27]. The placement of the groups and the dynamics of the hosts in the geographic space have also been completely re-designed. Furthermore, this paper presents a thorough evaluation of the model and a comparison with real traces, which is not presented in [25].

The paper has the following structure: Section 2 contains a definition of social network and illustrates some of the results offered by social network theory. Section 3 shows how these results can be used to design a social network founded mobility model. Section 4 illustrates the results of the evaluation of the model based on the comparison with real traces; some simulation results about the impact of the proposed mobility model on the performance of the AODV and DSR protocols are also discussed. In Section 5 we compare the proposed mobility model with the current state of the art and we outline our current research directions. Section 6 concludes the paper, summarizing the original contribution of our work.

2. SOCIAL NETWORKS

A social network describes a set of people (or groups of people) with some pattern of contact or interaction among each others [36]. Research studies in the area of social networks started in the 1920s [9]. However, the first significant quantitative results were presented by Rapoport [33] and his colleagues in the 1950s and 1960s in a series of papers in which they analyzed the statistics of epidemic diffusion in populations characterised by different social structures.

Whilst this was pioneering exploratory work, it was not rigorous from a scientific point of view. However, in that period, a renewed interest in graph theory led to the definition of the so-called random graphs by Paul Erdős and Alfred Rényi [8]. This, then, was the beginning of the complex networks research area, investigating properties such as their topology, average diameter and degree of connectivity, as well as the presence of clusters in networks.

In the recent years, various types of networks (such as the Internet, the World Wide Web and biological networks) have been investigated by many researchers especially in the statistical physics community. Theoretical models have been developed to reproduce the properties of these networks, such as the so-called small worlds model proposed by Watts and Strogats [41] or various scale-free models[1] [29, 40]. Excellent reviews of the recent progress in complex and social networks analysis may be found in [1] and [29].

However, as discussed by Newman and Park in [31], social networks appear to be fundamentally different from other types of networked systems. In particular, even if social networks present typical small-worlds behaviour in terms of the average distance between pairs of individuals (the so-called *average path length*), they shows a greater level of clustering. In particular, in [31] the authors observe that the level of clustering seen in many non-social systems is no greater than in those generated using pure random models. Instead in social networks, clustering appears to be far greater than in networks based on stochastic models. The authors suggest that this is strictly related to the fact that humans usually organize themselves into *communities*. Examples of social networks used for these studies are rather diverse and include, for instance, networks of coauthorships of scientists [28] and the actors in films with Kevin Bacon [41].

Many mathematical models have been proposed in the recent years to generate synthetic social networks [41] that show the same properties of real ones. We will use these results in order to generate realistic social networks structures that are one of the fundamental input of the proposed mobility model.

3. DESIGN OF THE MOBILITY MODEL

In this section we show how we designed a mobility model which is founded on the results of social network theories briefly introduced above. Firstly, we describe how we represent the social network. Then, we present how we identify communities and groups in the network and how the communities are associated to a geographical space. Our observation here is that people with strong social links are likely to be geographically colocated often or from time to time. The next stage is to devise a model for the movements of the nodes, that, again, has to mirror the strength of social relationships. We argue that individuals with strong social relationships move towards (or within) the same geographical area. We will then show that the emergent movement patterns provides a good approximation of real ones.

3.1 Modelling Social Relationships

One of the classic ways of representing social networks are *weighted graphs*. Each node represents one person. The weights associated with each edge of the network is used to model the strength of the interactions between individuals. An example of social network is represented in Figure 1.

It is our explicit assumption that these weights, which are expressed as a measure of the strength of social ties, can also be read as a measure of the likelihood of geographic colocation, though the relationship between these quantities is not necessarily a simple one, as will become apparent. We model the degree of social interaction between two people using a value in the range $[0, 1]$. 0 indicates no interaction; 1 indicates a strong social interaction[2].

As a consequence, the network in Figure 1 can be represented by the 10×10 symmetric matrix $\mathbf{M}$ showed in Figure 2. We refer to the matrix represented social relationships as *Interaction Matrix*.

$$\mathbf{M} = \begin{bmatrix} 1 & 0.76 & 0.64 & 0.11 & 0.05 & 0 & 0 & 0.12 & 0.15 & 0 \\ 0.76 & 1 & 0.32 & 0 & 0.67 & 0.13 & 0.23 & 0.45 & 0 & 0.05 \\ 0.64 & 0.32 & 1 & 0.13 & 0.24 & 0 & 0 & 0.15 & 0 & 0 \\ 0.11 & 0 & 0.13 & 1 & 0.54 & 0.83 & 0.57 & 0 & 0 & 0 \\ 0.05 & 0.67 & 0.24 & 0.54 & 1 & 0.2 & 0.41 & 0.2 & 0.23 & 0 \\ 0 & 0.13 & 0 & 0.83 & 0.2 & 1 & 0.69 & 0.15 & 0 & 0 \\ 0 & 0.23 & 0 & 0.57 & 0.41 & 0.69 & 1 & 0.18 & 0 & 0.12 \\ 0.12 & 0.45 & 0.15 & 0 & 0.2 & 0.15 & 0.18 & 1 & 0.84 & 0.61 \\ 0.15 & 0 & 0 & 0 & 0.23 & 0 & 0 & 0.84 & 1 & 0.65 \\ 0 & 0.05 & 0 & 0 & 0 & 0 & 0.12 & 0.61 & 0.65 & 1 \end{bmatrix}$$

Figure 2: Example of an Interaction Matrix representing a simple social network.

The generic element $m_{i,j}$ represents the interaction between two individuals i and j. We refer to the elements of the matrix as the *interaction indicators*. The diagonal elements represent the relationships that an individual has with himself and are set, conventionally, to 1. In Figure 1, we have represented only the link associated to a weight equal to or higher than 0.25.

The matrix is symmetric since, to a first approximation, interactions can be viewed as being symmetric. It is, however, worth underlining that we are using a specific measure of the strength of the relationships. It is probable that by performing psychological tests, the importance of a relationship, such as a friendship, will be valued differently by the different individuals involved; in our modelization, this would lead to an asymmetric matrix. We plan to investigate this issue further in the future.

The Interaction Matrix is also used to generate a *Connectivity Matrix*. From the matrix $\mathbf{M}$ we generate a binary matrix $\mathbf{C}$ where a 1 is placed as an entry c_{ij} if and only if

[1]Scale-free networks are characterized by a degree distribution that shows the following power-law tail shape:

$$P(k) = k^{-\gamma}$$

A function $f(x)$ is scale-free if it remains unchanged to within a multiplicative factor under a re-scaling of the independent variable x (i.e., it has a power-law form) [29].

[2]It is worth noting that these indicators are *not* a measure of the subjective importance of the relationships, such as family ties or friendships. Let us consider the case of a person working in a town that is different from the one in which his parents live. In this case, the social relationship is strong from a genealogical (and affective) point of view, but is weak if we consider the likelihood of *direct* interaction between them. In other words, in our model, this relationship will be modelled using a low value. An example of strong social interaction may be the case of two colleagues sharing the same office.

$m_{i,j}$ is greater than a specific threshold t. The Connectivity Matrix extracted by the Interaction Matrix in Figure 2 is showed in Figure 3. The idea behind this is to say two people are considered interacting only if their interaction value is greater than the threshold. In our case, we have a direct mapping to the graph in Figure 1, where we set the threshold equal to 0.25.

$$\mathbf{C} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Figure 3: Example of a Connectivity Matrix representing a simple social network.

The Interaction Matrix (and, consequently, the Connectivity Matrix) can be derived by available data (for example, from a sociological investigation) or using mathematical models that are able to reproduce characteristics of real social networks. As we will discuss in Section 4.2.2, the default implementation of our model uses the so-called Caveman model [41] for the generation of synthetic social networks with realistic characteristics (i.e, high clustering and low average path length). However, this is a customizable aspect and if there are insights on the type of scenario to be tested, a user-defined matrix can be inputed.

3.2 Detection of Community Structures

The simulation scenario is established by associating groups of hosts to certain area in geographical space. Therefore, after the definition of the social graph described above, groups, i.e., the highly connected set of nodes in the graph, need to be isolated. Fortunately, there are some algorithms which can be exploited for this purpose.

We use the algorithm proposed by Newman and Girvan in [30] to detect the presence of community structures in social networks. This algorithm is based on the calculation of the so-called *betweenness* of edges. The betweeneess provides a measure of the centrality of nodes. For example, considering two communities connected by few intercommunity edges, all the paths through the nodes in one community to nodes in the other must traverse one of these edges, that, therefore, will be characterised by a high betweenness. Intuitively then, one of the possible estimation of the centrality of an edge is given by the number of shortest (geodesic) paths between all pairs of vertices that run along it. In other words, the average distance between the vertices of the network has the maximum increase when the nodes with the highest betweenness are removed.

Therefore, in order to extract the communities from the network, nodes characterized by high values of centrality are progressively detected in following round. At each round, one of the edge of the host with the highest centrality is removed. The final result is a network composed of (isolated) groups of hosts (i.e., the communities).

The complexity of this algorithm is $O(mn^2)$, considering a graph with m edges and n vertices. The calculation of the shortest path between a particular pair of vertices can be perfomed using a breadth-first search in time $O(m)$ and there are $O(n^2)$ vertices. However, in [30], Newman and Girvan proposed a faster algorithm with a complexity equal to $O(mn)$. A coincise description of this algorithm for the calculation of the betweenness can be found in the appendix of this paper.

As we said, the algorithm can be run a number of times on the graph, severing more and more links and generating a number of isolated communities. However we need to derive a mechanism to stop the algorithm when further cuts would decrease the quality of the results: this would mean that we have reached a state when we have meaningful communities already. We adopted a solution based on the calculation of an indicator defined as *modularity* Q [30]. This quantity measures the proportion of the edges in the network that connect vertices within the same community minus the expected value of the same quantity in a network with the same community division but random connections between the vertices. If the number of edges within the same community is no better than random, the value of Q is equal to 0. The maximum value of Q is 1; such a value indicates very strong community structure. In real social networks, the value of Q is usually in the range $[0.3, 0.7]$. The analytical definition of the modularity of a network division is presented in Section B of the appendix.

At each run the algorithm severs one edge at a time and measures the value of Q. The algorithm terminates when the obtained value of Q is less than the one we have obtained in the previous edge removal round. This is motivated by the fact that Q is a monotic function and usually presents one or, at maximum, but much more rarely, two local peaks: therefore, we can stop when the first local peak is reached. This is clearly an approximation since the value of the other possible local peak (if exists) may be higher, but it has been observed that the quality of the division that we obtain is in the vast majority of the cases very good [30]. Also, by adopting this technique, we considerably simplify the computational complexity of the algorithm.

In order to illustrate this process, let us now consider the social network taken as example represented in Figure 1. Three communities (that can be represented by sets of hosts) are detected by running the algorithm: $C_1 = \{A, B, C\}$, $C_2 = \{D, E, F, G\}$ and $C_3 = \{H, I, L\}$. Now that the communities are identified given the matrix, there is a need to associate them with a location.

3.3 Placement of the Communities in the Simulation Space

After the communities are identified, each of them is randomly associated to a specific location (i.e., a square) on a grid[3]. We use the symbol $S_{p,q}$ to indicate a square in position p, q. The number of rows and columns are inputs of the mobility model.

Going back to the example, in Figure 4 we show how the communities we had identified could be placed on a 3x4 grid (the dimension of the grid is configurable by the user and influences the density of the nodes in each square). The three communities C_1, C_2, C_3 are placed respectively in the grid in the squares $S_{a,2}$, $S_{c,2}$ and $S_{b,4}$.

Once the nodes are placed on the grid, the model is estab-

[3]A non random association to the particular areas of the simulation area can be devised, for example by deciding predefined *areas of interest* corresponding for instance to real geographical space. However, this aspect is orthogonal to the work discussed in this paper.

lished and the nodes move around according to social-based attraction laws as explained in the following.

3.4 Dynamics of the Mobile Hosts

Each node is assigned to a *goal*. The goal is simply a point on the grid which acts as final destination of movement like in the Random Way Point model, with the exception that the selection of the goal is not as random. Consequently, a node is also associated to a certain square in the grid. We say that a host i belongs to a square $S_{p,q}$ if its goal is inside $S_{p,q}$.

3.4.1 Selection of the first goal

When the model is initially established, the goal of each host is randomly chosen inside the square associated to its community (i.e, the first goals of all the hosts of the community C_1 will be chosen inside the square $S_{a,2}$).

3.4.2 Selection of the subsequent goals

When a goal is reached, the new goal is chosen according to the following mechanism. A certain number of hosts (that may be also equal to 0) is associated to each square $S_{p,q}$ at time t. Each square (i.e., place) exterts a certain *social attractivity* for a certain person. The social attractivity of a square is a measure of its importance in terms of the social relationships for the individual taken into consideration. The social importance is calculated by evaluating the strength of the relationships with the people that are moving towards that particular square (i.e., with the individuals that have a current goal inside that particular square). More formally, given $C_{S_{p,q}}$ the set of the hosts associated to the square $S_{p,q}$, we define the social attractivity of that square towards the host i SA_{p,q_i} as follows

$$SA_{p,q_i} = \frac{\sum_{\substack{j=1 \\ j \in C_{S_{p,q}}}}^{n} m_{i,j}}{w}$$

where w is the cardinality of $C_{S_{p,q}}$ (i.e., the number of hosts associated to the square $S_{p,q}$). In other words, the social attractivity of a square in position (p, q) towards an individual i is defined as the sum of the interaction indicators that represent the relationships between i and the other hosts that belong to that particular square, normalized by the total number of hosts associated to that square.

The new goal is then randomly chosen inside the square characterised by the highest social attractivity; it may be again inside the same square or in a different one. New goals are chosen inside the same area when the input social network is composed by loosely connected communities (in fact, in this case, hosts associated with different communities have, in average, weak relationships between each others). On the other hand, a host may be attracted to a different square, when it has strong relationships with both communities. In other words, from a graph theory point of view, the host is located between two (or more) clusters of nodes in the social network[4].

Let us suppose, for example, that host A has reached its first goal inside the square $S_{a,2}$. The new goal is chosen by

[4]This is usually the case of hosts characterised by a high betweenness that, by definition, are located *between* two (or more) communities.

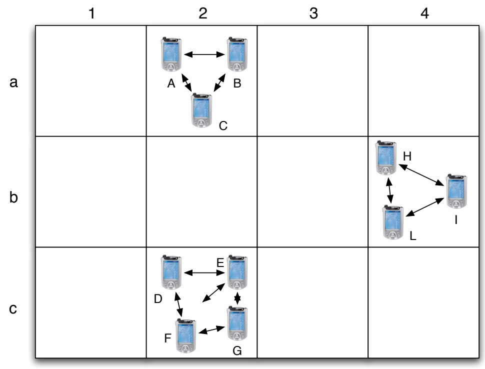

Figure 4: Example of initial simulation configuration.

calculating the social attractivities of all the squares that composed the simulation space. Let us suppose that the square $S_{c,2}$ now exterts the highest attractivity (for example, because a hosts with strong relationship with node A has joined that community). Therefore, the new goal will be selected inside that square.

3.5 Social Network Reconfigurations and their Effects on the Dynamics of Mobile Hosts

Like in everyone's life, the day movement are governed by different patterns of mobility which depend on the people we need to interact with. For example, most human beings spend a part of their day at work, interacting with colleagues, and another part at home with their families. In order to model this, we allow the association of different social networks to different periods of a simulation.

Periodically, the social networks at the basis of the mobility model can be changed. The interval of time between changes is an input of the model. When the reconfiguration of the underlying social network happens, nodes are assigned to the new communities that are detected in the network using the algorithms described in Section 3.2. Communities are then randomly associated to squares in the simulation space. New goals are then assigned to the mobile nodes. Goals are chosen inside the square of the grid to which the community they belong to is assigned. Notice that the nodes are not relocated instantaneously, but they will move towards their destination gradually. The nodes start moving towards the geographical region where other nodes that have strong interactions with them will converge. This mirrors the behaviour, for instance, of commuters who travel home every evening to join their families.

4. EVALUATION AND DISCUSSION

In order to evaluate our model we have performed a number of tests, in particular we have taken real mobility traces collected by Intel Research Laboratory in Cambridge. We have then tested our model using realistic social networks and compared the mobility patterns with the Intel traces. We have also compared the performance of AODV [32] and DSR [19] using the Random Way Point and our Community based mobility models. In this section, we will present and discuss the results of our simulations comparing them with these data from real scenarios.

4.1 Implementation of the model

We implemented a movement patterns generator that produce primarily traces for the ns-2 simulator [23], one of the most popular in the ad hoc network research community. However, the generator is also able to produce traces in a XML meta-format that can be parsed and trasformed into other formats (for example, by using XSLT) such as the one used by GlomoSim [42].

The model is available for downloading at the following URL: *[omitted for blind review]*.

4.2 Validation of the Model using Real Movement Traces

In this section, we present a comparison of the properties of the movement patterns generated by our mobility model with those of the real traces provided by Intel Research Laboratory in Cambridge. The description of these measurement exercise is presented in [6]. In this paper, the authors also compare their results with other publicly available data sets provided by McNett and Voelker from University of California at San Diego [24] and by Henderson et alii from Dartmouth College [12] showing evident similarities between the patterns movements collected by the three different groups. For this reason, we decided to compare the traces obtained by using our mobility model only with the data provided by the researchers in Cambridge[5].

4.2.1 Description of the Data Sets

The traces were collected by Intel researchers using iMotes (a modified version of the Berkeley Motes) [2] equipped with Bluetooth support. The iMotes were then given to members of the staff of Intel Research Laboratory and University of Cambridge. The iMotes were packed in keyfobs in order to make sure that people carried them around.

Each iMotes logged contacts data in a flash memory using the standard Bluetooth Baseband layer inquiry procedure. Every contact was stored as a tuple composed of three fields, the MAC address of the other device, the start and the end of the interval of time of the contact. Every iMotes collected information not only about the other samplers but also related to the other Bluetooth devices that they were in reach of them.

The iMotes were programmed to perform an inquiry for 5 seconds every $2+\Delta$ minutes with Δ randomly chosen in the range $[-12, 12]$ seconds. This correction was introduced to avoid undesired synchronization effects, i.e., to avoid that the iMotes performed inquiries at the same time. In fact, iMotes are not able to perform and reply to inquiries contemporaneously. Between inquiries, the iMotes go in sleep mode, where they are still able to provide replies.

4.2.2 Description of the Simulation

We tested our mobility model using several runs generating different mobile scenarios and we compared it with the real movement patterns provided by Intel and synthetic traces generated using a Random Way Point model.

We tested our model considering a scenario composed of 100 hosts in a simulation area of 5 $km \times 5$ km, divided into 625 squares of 200 m (i.e., the numbers of rows and columns were set to 25). We choose a relatively large simulation scenario with a low population density in order to differentiate the results from those obtained with a Random Way Point model. In fact, in small simulation areas, the limited possible movements in the area and the higher probability of having two nodes in the same transmission range may affect the simulation results introducing side-effects that are not entirely due to the mobility model.

We also assumed that each device is equipped with an omnidirectional antenna with a transmission range of 250 m, modeled using a free space propagation model. The speeds of the nodes were randomly generated according to a uniform distribution in the range $[1-6]$ m/s. The duration of the simulation is one day and the reconfiguration interval is equal to 8 hours. These values have not been chosen to reproduce exactly the movements described by the traces provided by Intel. We were more interested in observing if similar patterns could be detected in synthetic and real traces. In other words, our goal has mainly been to verify whether the movement patterns observed in Intel traces are reproduced by our mobility model.

A key aspect of the initialization of our model is the selection of the social network in input. We implemented a generator of social networks using the so-called Caveman Model proposed by Watts [41]. The network is built starting from K fully connected graphs (representing communities living in isolation, like primitive men in caves). Every edge is rewired to point to another cave with a certain probability p. Figure 5.a shows an initial network configuration composed by 3 disconnected communities (*caves*) composed by 5 individuals; a possible social network after random rewiring is represented in Figure 5.b.

Individuals of one cave are closely connected, whereas populations belonging to different caves are sparsely connected. Therefore, the social networks generated using this model are characterized by a high clustering coefficient and low average path length. It has been proved that this model is able to reproduce social structures very closed to real ones [41]. We generated social networks with different rewiring probabilities, also considering the case of disconnected communities (i.e.,$p = 0$).

We also implemented a movement patterns generator based on the Random Way Point model. We generated traces with the same simulation scenarios in terms of size of the area and characteristics of the mobile devices, with hosts that move with a speed uniformly distributed in the range $[1-6]$ m/s and stop time equal to $[1-10]$ m/s.

We repeated the experiments using a number of runs sufficient to achieve a 10% confidence interval.

4.2.3 Simulation Results

We analyzed two properties of the movement patterns, the contact duration and the inter-contacts time. We use the same definitions of the authors of [6] in order to compare our results with those presented in their work. We define *contact duration* as the time interval for which two devices can communicate when they are in the same radio range. We also define *inter-contact time* as the time interval between two contacts. These indicators are particularly important in ad hoc networking and in particular for *opportunistic mobile networks*, such as delay tolerant mobile ad hoc networks [26, 20]. In fact, inter-contacts times define the frequency and

[5]As said in the introduction of this paper, the measurement exercise was also repeated among the partecipants of InfoCom 2005 [16]. In our comparison, we used the traces related to the first experiment. However, the results obtained in the two different studies show remarkable similarities.

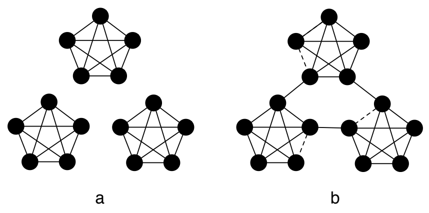

Figure 5: Generation of the social network in input using the Caveman model: (a) initial configuration with 3 disconnected 'caves'. (b) generated social network after the rewiring process.

the probability of being in contact with the recipient of a packet or a potential carrier in a given time period.

Figure 6 shows the comparison between the inter-contacts time and the contacts duration cumulative distributions[6] using log-log coordinates. These distributions are extracted from the real and synthetic traces generated by the Random Way Point (indicated with RWP) and our Community based mobility model (indicated with CM) with different rewiring probabilities p.

With respect to the inter-contacts time, our traces (excluding the case with $p = 0$ that we will discuss separately) shows an approximate power law behaviour for a large range of values like those extracted from Intel data. A similar pattern can be observed in UCSD and Dartmouth traces [6]. The cumulative distribution related to Random Way Point instead shows a typical exponential distribution. The same behaviour can be observed for the traces generated using our Community based mobility model with a probability of rewiring equal to 0. In fact, in this case, the only movements of the hosts outside the assigned square happen when a reconfiguration takes place (i.e., a new generation of the social networks takes and a consequent new assignment to different squares in the grid are performed).

As far as the contacts time distribution is concerned, it is possible to observe a power-law behaviour for a much more limited range of values and in general with a lower angular coefficient of the interpolating line. It is worth noting that the traces from Dartmouth College and UCSD show a power-law distribution with different angular coefficients. It seems that data related to different scenarios are characterized by different types of power-law distribution. The cumulative distribution of the contacts time extracted by the traces generated by our mobility model is typically power-law for a large set of values.

By plotting the same distributions using semi-log coordinates (see Figure 7), the differences between the curves corresponding to real traces and those generated using the Random Way Point mobility model are even more evident. The exponential nature of the cumulative distribution of the inter-contacts time[7] extracted by the latter is clearly reflected by the approximated straight line that is shown in the figure.

Figure 8.a and 8.b show the influence of the speed respectively on the cumulative distributions of the inter-contacts time and contacts duration. We simulated scenarios with host with a speed uniformly distributed in the range $[1-6]$, $[1-10]$ and $[1-20] m/s$. It is interesting to note that the cumulative distributions related to all these scenario can be approximated with a power-law function for a wide range of values.

The impact of the density of the population in the simulation scenario is presented in Figure 11. We simulated scenarios composed by 100, 200, 300 nodes with a starting number of groups for the Caveman model respectively equal to 10, 20, 30 and a rewiring probability equal to 0.2. Also in these scenarios, the inter-contacts time and contacts duration distributions follow a similar pattern. As discussed previously, our aim was not to reproduce exactly the traces provided by Intel. However, quite interestingly we observe that the inter-contacts time distribution lie in between the curves representing the scenario composed of 100 and 200 nodes. The number of nodes recorded in the Intel experiments was in fact 140. Instead, the contacts duration distribution is bounded by the curves extracted by these two synthetic traces for a smaller range of values. Finally, in Figure 8 we consider a scenario composed of 100 hosts connected by a social network generated using different initial number of groups (i.e., caves) as input for the Caveman model (with a re-wiring probability equal to 0.1). By varying the number of groups, the density of the squares of the grid changes. It is interesting to note that the power-law patterns can be observed in all the scenarios also with a large number of small initial groups.

4.3 Influence of the Choice of the Mobility Model on Routing Protocol Performance

4.3.1 Simulation Description

We also tested the mobility model in case of dense networks. Using ns-2, we simulated a scenario composed of 50 hosts and we compared the performance in terms of delivery ratio of the AODV [32] and DSR [19] protocols We used a $1000m \times 1000m$ simulation area with a maximum node transmission range equal to $250m$. We chose the two-ray pathloss model as propagation model and at the MAC layer, the IEEE 802.11 DCF protocol was used with a bandwidth equal to 2 Mbps. We started 10 random sessions using CBR traffic with data packet size and sending rate respectively equal to 512 bytes and 4 packets/second. The simulation time was equal to 2 hours.

We studied the influence of the speed on the performance comparing the results obtained by using the Random Way Point model and the Community based mobility model presented in this paper. Every node in the simulation is moving at the same speed. With respect to the Random Way Point model, the stopping time are chosen randomly in the interval $[1-10] m/s$. As far as our mobility model is concerned, the reconfiguration interval was set to 1 hour. The social network in input was generated with the Caveman model with 5 groups of 10 individuals and a re-wiring probability equal to 0.1. The simulation scenarios was divided into a $5x5$ grid.

Also for this set of experiments we performed a number of runs sufficient to achieve a confidence interval of 10% error.

[6]Cumulative distributions are generally used instead of frequency distributions to avoid the issues related to the choice of the bins of the plot. It is possible to prove that if a set of data shows a power-law behaviour using a frequency histogram, its cumulative distribution also follows the same pattern.

[7]This behaviour has been theoretically studied and predicted by Sharma and Mazumdar in [37].

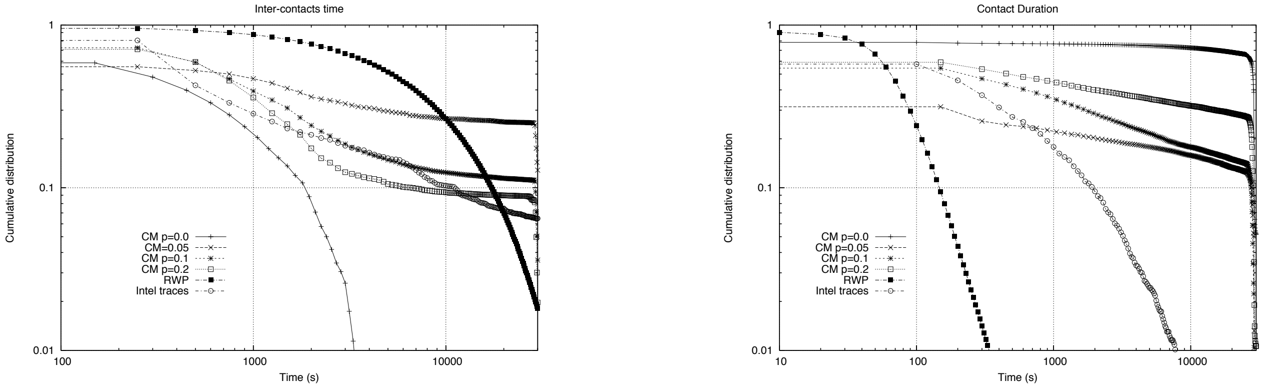

Figure 6: Comparison between synthetic and real traces (log-log coordinates) : (a) cumulative distribution of inter-contacts time in seconds; (b) cumulative distribution of contacts duration in seconds.

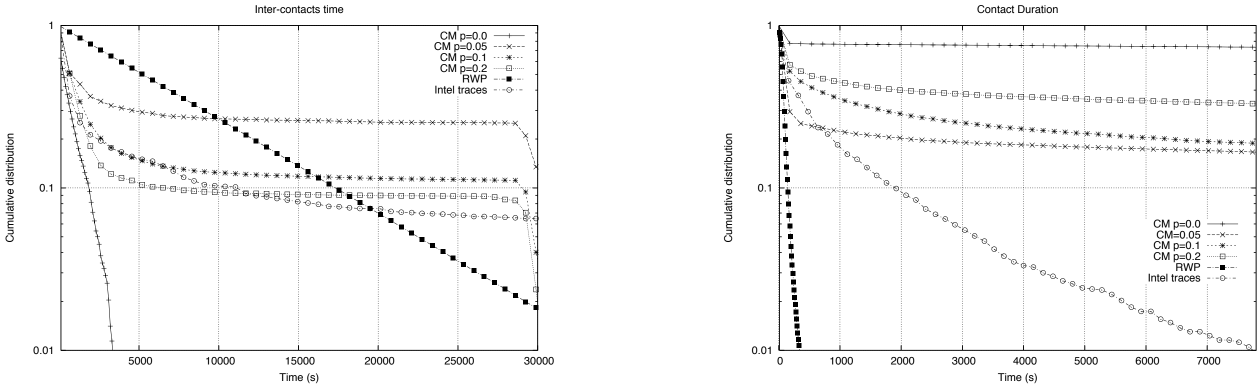

Figure 7: Comparison between synthetic and real traces (semi-log coordinates): (a) cumulative distribution of inter-contacts time in seconds; (b) cumulative distribution of contacts duration in seconds.

4.3.2 Simulation Results

Using the Random Way Point mobility model, as expected and confirming the results obtained by the authors of these protocols, the delivery ratio decreases as the speed increases. Instead, using our model, the decrease of the delivery ratio is less evident, since the emerging structure is composed of groups of hosts moving in limited areas (i.e., the square of the grids) that are 'bridged' by hosts roaming among them. In other words, the movement of most of the hosts is constrained in geographical terms and then topology changes are less frequent than in the case of a pure random model.

It is also interesting to note that in case of fixed hosts (i.e., with a speed equal to 0), the delivery ratio that we obtained using our mobility model is lower than in the scenarios with a speed greater than 0, since in the former case we may have disconnected communities, whereas in the latter hosts move between communities, providing a link between them.

5. DISCUSSION

5.1 Related Work

Many mobility models have been presented with the aim of allowing scalability testing of protocols and algorithms for mobile ad hoc networking. A comprehensive review of the most popular mobility models used by the mobile ad hoc research community can be found in [5]. However, it is interesting and, at the same time, surprising to note that even the best solutions and approaches have only been tested using completely random models such as the Random Way-Point model, without grouping mechanisms. A more refined approach used a simple groups mobility model which still had a large random component in the way groups were created and moved [14]. The almost pervasive adoption of such models has generated a considerable amount of work that is predicated on the reasonableness of random mobility models.

The work most directly related to ours can be found in [13]. This model is predicated upon similar assumptions, but is considerably more limited in scope. In that model hosts are statically assigned to a particular group during the ini-

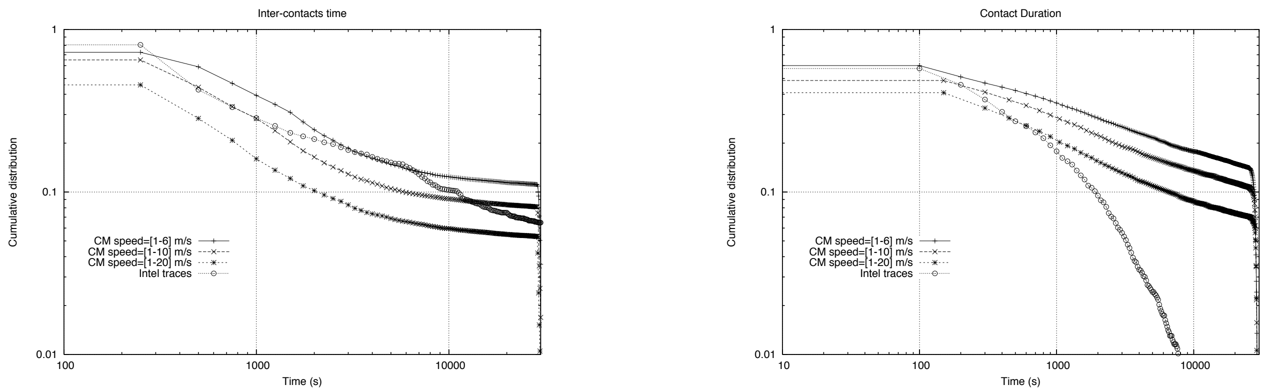

Figure 8: Influence of the hosts speed: (a) cumulative distribution of inter-contacts time in seconds; (b) cumulative distribution of contacts duration in seconds.

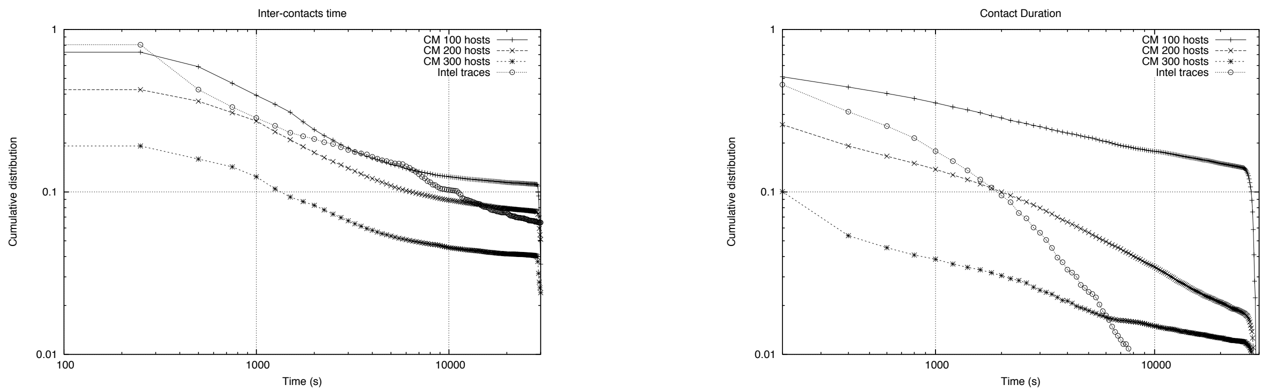

Figure 9: Influence of the density of population: (a) cumulative distribution of inter-contacts time in seconds; (b) cumulative distribution of contacts duration in seconds.

tial configuration process, whereas our model accounts for movement between groups. Moreover, the authors claim that mobile ad hoc networks are scale-free, but the typical properties of scale-free networks are not exploited in the design of the model presented by the authors. The scale-free distribution of mobile ad hoc networks is still not proven in general, since practical measurements are not currently available. Scale-free properties are strictly dependent on the movements of hosts and therefore are dependent on the actual simulated scenarios/applications [11]. With respect to this work we allow the setting of the initial social network, which conditions the movement patterns, this allows different kind of networks to emerge, including small world and scale free.

In the recent years, many researchers have tried to refine existing models in order to make them more realistic. In [17], a technique for the creation of more realistic mobility models that include the presence of obstacles is presented. The specification of obstacles is based on the use of Voronoi graphs in order to derive the possible pathways in the simulation space. This approach is orthogonal to ours; this would be an interesting extension of the model as discussed in the next section.

Tuduce and Gross in [39] present a mobility model based on real data from the campus wireless LAN at ETH in Zurich. They use a simulation area divided into squares and derive the probability of transitions between adjacent squares from the data of the access points. It is interesting to note that also in this case the session duration data follow a power-law distribution. This approach can be a refined version of the Weighted Waypoint Mobility Model [15], based on the probability of moving between different areas of a campus using a Markov model. Moreover, Tuduce and Gross' model represent the movements of the devices in an infrastructure-based network and not ad hoc settings. In [22], the authors tried to reproduce the movements of pedestrians in downtown Osaka by analysing the characteristics of the crowd in subsequent instants of time and maps of the city using an empirical methodology. In general, the main goal of these works was to try to reproduce the specific

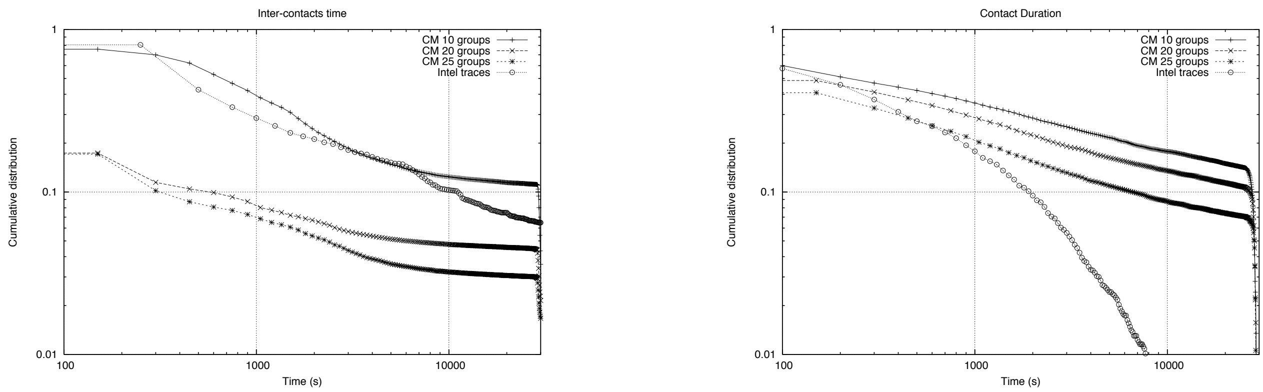

Figure 10: Influence of the number of initial number of groups in input to the network generator based on the Caveman model: (a) cumulative distribution of inter-contacts time in seconds; (b) cumulative distribution of contacts duration in seconds.

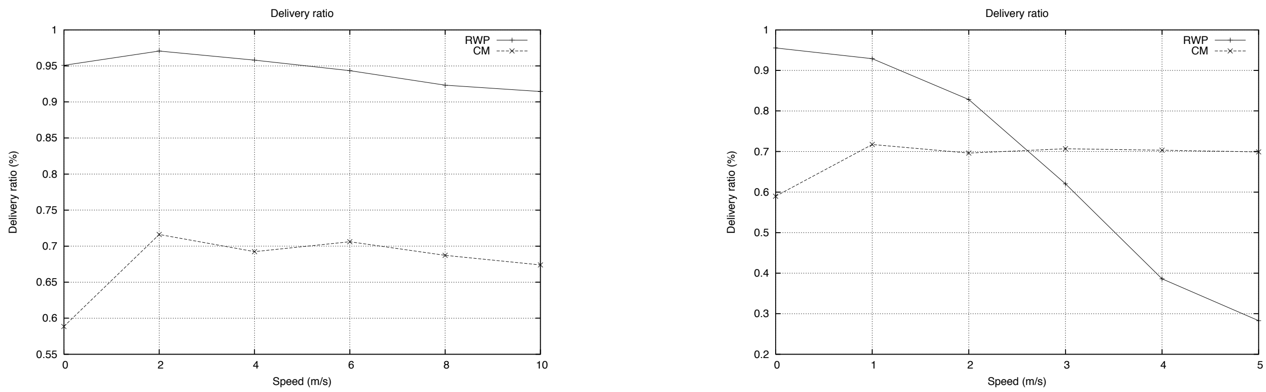

Figure 11: Influence of the mobility model on the protocol performance (delivery ratio vs speed): (a) AODV; (b) DSR.

scenarios with a high degree of accuracy. We focus instead on the cause of these movements, trying to capture the social dimensions that lead to general emergent human movement patterns.

Some interesting studies have been recently carried out on the connectivity of ad hoc networks with respect to complex networks theory. Glauche et al. in [11] discuss some network properties using percolation theory [38], that is, an application of complex networks theory derived by the investigation of physical phenomena such as phase transitions in molecular lattices. In [35], the authors present mathematical results about the possible emergence of scale-free structures in ad hoc networks. However, the authors consider only *fixed* ad hoc networks (such as peer-to-peer networks), without analysing the influence of movement in the definition of their model.

5.2 Possible Improvements of the Model and Current Research Directions

We believe that a number of possible features can be added to the mobility model presented in this paper in order to increase its realism. Many researchers in the ad hoc community have been focussing on different aspects of this problem. We believe that some of the techniques are orthogonal to our work and can be integrated in our model. We plan to explore these possible refinements of the model in the future. More specifically, in our opinion, some of the most important improvements can be summarized as follows:

- **Non random assignment of the nodes to the geographical locations** A possible improvement of the model may be the assignment of the communities based on real mapping between groups of people and geographical locations (such as students moving around lecture rooms and halls in a campus, etc.). An example of this kind of mobility models is [15]. In the current implementation, the generation of the social networks is based on the mathematical model described above and the placement of the communities is random. This allows for multiple runs with different automatically generated social networks and mobile scenarios. However, the current implementation can be easily modified and replaced by a custom initialization of the simulation settings.

- **Movement determined by pre-defined trails** Many existing mobility models are based on the definition of *trails* or *paths* that are used to define the movements of the mobile nodes in the simulation scenarios. Examples are the Manhattan model [5] or other models used to simulate protocols and systems for vehicular ad hoc networks [34]. In these models hosts move between different locations following precise paths that represent roads or motorways. We plan to study the effects of the introduction of pre-defined trails in our model, in particular to characterize the movements between different communities (i.e., squares of the grid) with better accuracy.

- **Presence of obstacles** As discussed in the previous subsection, in [17] Jardosh et alii. proposes a modified version of the Random Mobility Model that allows for the insertion of obstacles in the simulation space. The definition of obstacles, can also be integrated in our mobility model easily.

Finally, we plan to study the connectivity of the generated mobile networks also in relation to the social networks given in input also using results from graph theory studies [3].

6. CONCLUSIONS

We have presented a new mobility model which is based on social network theory and is predicated on the assumption that mobility patterns are driven by the fact that devices are carried by humans and then the movements are strongly affected by the relationships between them.

The paper has shown how the mobility model is generated, its implementation and an evaluation based on the comparison between our approach, existing random mobility models and real movement traces. We have shown that our mobility model generates traces that present characteristics similar to real ones, in terms of inter-contacts time and contacts duration.

Acknowledgments

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APPENDIX

A. BETWEENNESS OF A NODE

The betweenness of a node in a graph is defined as the total number of shortest paths between pairs of nodes that pass through it [10]. The steps of the algorithm proposed by Newman are the following:

- First of all, the shortest path to a vertex j from every vertices in the network are calculated by using the following algorithm:
  1. Assign to vertex j a distance d equal to 0;
  2. For each vertex i whose assigned distance is i, follow each attached edge to the vertex k at its other end and if k has not already been assigned a distance, assign it distance $d+1$. i is declared a predecessor of k;
  3. Increment the value of the distance d by 1;
  4. Repeat from step 2 until there are not unassigned vertices left.
- A variable b_i is assigned to every vertex of the network. The variables are initialized to 1.
- Considering the vertices i in order from the farthest to the nearest, the value of b_i is added to the corresponding variable on the predecessor vertex of i. If i has more than one predecessor, then b_i is divided equally by them.
- The resulting values of b_i represent the number of geodesic paths to vertex j that run through each vertex of the lattice. To calculate the betweenness for all paths, the b_i are added to a variable b_i' that is maintained for each variable and the entire calculation is repeated for all the vertices. The final value of b_i' represents the betweenness of the vertex i.

The algorithm is presented and discussed in more details in [30].

B. MODULARITY

Let us consider a division of a network into k communities. We define a $k \times k$ symmetric matrix $\mathbf{e}$ the elements e_{ij} of which is the proportion of all edges in the network that link vertices in community i to vertices in community j. The trace of this matrix $Tr\ \mathbf{e} = \sum_i e_{ii}$ gives the proportion of edges in the network that connect vertices in the same community. A good division into communities should have a high value of this trace, meaning that a good portion of the edges of the network is of edges "inside" a community. However, this is not sufficient to judge the quality of the division. In fact, the case of one single community corresponds to the case of $Tr\ \mathbf{e} = 1$.

Therefore, we define the rows sums $a_i = \sum_j e_{ij}$ which represents the proportion of edges that connect to vertices in community i. Thus, it is possible to define the *modularity* Q of a network division as follows:

$$Q = \sum_i (e_{ii} - a_i^2) = Tr\ \mathbf{e} - ||\mathbf{e}^2||$$

where $||\mathbf{e}^2||$ the sum of all the elements of the matrix $\mathbf{e}^2$.

More details can be found in [30].